Effect of Couple Stresses on the MHD of a Non-Newtonian Unsteady Flow between Two Parallel Porous Plates

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In this paper the MHD of a Non-Newtonian unsteady flow of an incompressible fluid under the effect of couple stresses and a uniform external magnetic field is analysed by using the Eyring Powell model. In the first approximation the solution is obtained by using the Mathematica computational program with assuming a pulsatile pressure gradient in the direction of the motion. In the second order approximation a numerical solution of the non-linear partial differential equation is obtained by using a finite difference method. The effects of different parameters are discussed with the help of graphs in the two cases.

Key words: Magnetohydrodynamic (MHD); Couple Stresses; Non-Newtonian; Porous Plates.

An important class of fluids differs from Newtonian fluids in that the relationship between the shear stress and the flow field is more complicated. Such fluids are non-Newtonian. Examples include various suspensions such as coal-water or coal-oil slurries, food products, inks, glues, soaps, polymer solutions, etc.

Studies of unsteady magnetohydodynamic (MHD) flows of non-Newtonian fluids have been made for planer porous walls and in the zero-induction approximation. MHD flows require time to obtain the steady velocity distribution. The influence of the magnetic field on the starting phase was demonstrated for the Hartmann flow [1], assuming a small magnetic Reynolds number and a constant pressure gradient in the direction of the flow. Approximate solutions were obtained by Yen [2], when the MHD Hartmann flow is affected by a periodic change of the pressure gradient. Shaikadz and Megahed [3] studied the problem of an unsteady MHD flow assuming constant pressure, and an exact solution was obtained when the upper wall is moving with a time dependent velocity, where the two walls were not porous.

In [4] an unsteady MHD non-Newtonian flow between two parallel fixed porous walls was studied using the Eyring Powell model [5], and in first approximation an exact solution of the velocity distribution was obtained if the pressure gradient in the direction of the motion is an arbitrary function of time. In second approximation a numerical solution was obtained when the pressure gradient is constant. A non-Newtonian fluid flow between two parallel walls, one of them moving with a uniform velocity under the action of a transverse magnetic field, was studied in [6].

The present paper treats the flow of a pulsatile non-Newtonian incompressible and electrically conducting fluid in a magnetic field. Possible applications of these calculations are the flow of oil under ground, where there is a natural magnetic field and the earth is considered as a porous solid, and the motion of blood through arteries where the boundaries are porous.

Couple stresses are the consequence of assuming that the mechanical action of one part of a body on another across a surface is equivalent to a force and moment distribution. In the classical nonpolar theory, moment distributions are not considered and the mechanical action is assumed to be equivalent to a force distribution only. The state of stress is measured by a stress tensor τ_{ij} and a couple stress tensor M_{ij}. The purpose of the present paper is to investigate the effect of couple stresses on the flow by obtaining the effect of the couple stress parameter besides other parameters entering the problem on the velocity distribution. The field equations are [7]:

The continuity equation $\dot{\rho}+\rho v_{i,i}=0$, Cauchy's first law of motion $\rho a_i = T_{ji,j}+\rho f_i$, and Cauchy's second law of motion $M_{ji,j}+\rho \ell_i + e_{ijk}T_{jk}=0$, where ρ is the density of the fluid, v_i are the velocity components, a_i the components of the acceleration, T_{ij} is the second order stress tensor, M_{ij} the second order couple stress tensor, f_i the body force per unit volume, ℓ_i the body moment per unit volume and e_{ijk} the third order alternating pseudo tensor, which is equal to $+1$ or -1 if (i,j,k) is an (even or odd) permutation of $(1,2,3)$, and is equal to zero if two or more of the indices i,j,k are equal. Mindlin and Tiersten [8] obtained the constitutive equations for a linear perfectly elastic solid in the form

$$T_{ji,j} = T^{\mathrm{s}}_{ji,j} + T^{\mathrm{A}}_{ji,j},$$

where T^{s}_{ji} is the symmetric part of the stress tensor.

$$T^{\mathrm{s}}_{ji,j} = -P_{,i} + (\lambda+\mu)v_{j,ji} + \mu v_{i,jj} = -P_{,i} + (\tau_{ji})_{,j},$$

and $T^{\mathrm{A}}_{ji,j} = -2\eta\,\omega_{ji,jll} + \frac{1}{2}e_{ijk}(\rho\ell_k)_{,j}$, where τ_{ij} is the stress tensor in the nonpolar classical theory, ω_{ij} is the spin tensor, which is considered as a measure of the rates of rotation of elements in a certain average sense, and $\omega_{ij} = e_{ijk}\omega_k$, where ω_i is the vorticity vector. Since $\omega_i = \frac{1}{2}e_{ijk}v_{k,j}$, one has $\omega_{ij} = \frac{1}{2}(v_{j,i} - v_{i,j})$, and we can write

$$T^{\mathrm{A}}_{ji,j} = -\eta v_{i,jjll} + \eta v_{j,jill} + \frac{1}{2}e_{ijk}(\rho\ell_k)_{,j},$$

and the equation of motion becomes

$$\rho a_i = T^{\mathrm{s}}_{ji,j} + \eta(v_{j,j})_{,ikk} - \eta v_{i,jjkk} + \frac{1}{2}e_{ijsk}(\rho\ell_k)_{,j} + \rho f_i.$$

For incompressible fluids and if the body force and body moment are absent, the equations of motion reduce to $\rho a_i = T^{\mathrm{s}}_{ji,j} - \eta v_{i,jjkk}$, which, in vector notation, can be written as

$$\rho \boldsymbol{a}_i = -\nabla P + \nabla\cdot(\tau_{ij}) - \eta\,\nabla^4\mathbf{v}.$$

The last term in this equation gives the effect of couple stresses. Thus, for the effect of couple stresses to be present, $v_{i,rrss}$ must be nonzero. τ_{ij} represents the stress tensor in the case of the nonpolar theory of fluids. The Eyring-Powell model for describing the shear of a non-Newtonian flow is derived from the theory of rate processes. This model can be used in some cases to describe the viscous behavior of polymer solutions and viscoelastic suspensions over a wide range of shear rates. The stress tensor in the Eyring- Powell model for non-Newtonian fluids takes the form

$$\tau_{ij} = \mu\frac{\partial u_i}{\partial x_j} + \frac{1}{\beta}\sinh^{-1}\left(\frac{1}{c}\frac{\partial u_i}{\partial x_j}\right),$$

where μ is the viscosity coefficient. β and c are the characteristics of the Eyring-Powell model.

In the present work we study the problem of an unsteady MHD non-Newtonian flow between two parallel fixed porous plates, where the x and y axes are taken along and perpendicular to the parallel walls, respectively, and for the velocity $\mathbf{v} = (u(y,t), v(y,t), 0)$ in this case the continuity equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

which gives $\partial v/\partial y = 0$, i. e. $v = f(t)$. The function $f(t)$ is taken as a constant velocity V_0, which represents the velocity of suction or injection through the plate. Then the velocity tends to $u(y)$ only.

In hydromagnetic parallel flows of the type where the velocity is of the form $v_1 = u(y)$, $v_2 =$ constant, $v_3 = 0$, and these flows are assumed to be subjected to a uniform magnetic field B_0 in the positive y direction, the equation of motion becomes [7]

$$\rho\left(\frac{\partial u}{\partial t} + V_0\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}(\tau_{ij}) + \sigma B_0(E - uB_0) - \eta\frac{\partial^4 u}{\partial y^4}.$$

Neglecting the electric currents, i.e. E, the above equation becomes

$$\rho\left(\frac{\partial u}{\partial t} + V_0\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}(\tau_{ij}) + \sigma B_0^2 u - \eta\frac{\partial^4 u}{\partial y^4}.$$

The solution in case of the first approximation of the Eyring Powell model is obtained by using the Mathematica computational program with assuming a pulsatile pressure gradient in the direction of the motion. A numerical solution for the non-linear partial differential equation in second order approximation is obtained by using a finite difference method [9]. The effects of different parameters of the problem are discussed with the help of graphs in two cases.

Mathematical Formulation

Consider a non-Newtonian unsteady electrically conducting incompressible flow between two parallel porous walls situated a distant L apart under the effect of couple stresses. We take the x and y axes along and transverse to the parallel walls and assume a uniform magnetic field B acting along the y- axis. The fluid is injected into the lower wall at $y = 0$ and is sucked through the upper wall at $y = L$ with the uniform velocity V_0. The electric field is assumed to be zero. The induced magnetic field is assumed to be very small, and the electric conductivity σ of the fluid is sufficiently large. The governing equations are

$$\frac{\partial u}{\partial t} + V_0 \frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial P}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial y}(\tau_{xy}) - \frac{\sigma}{\rho} B_0^2 u - \frac{\eta}{\rho}\frac{\partial^4 u}{\partial y^4}, \tag{1}$$

$$\frac{\partial P}{\partial y} = 0, \tag{2}$$

where $u = u(y,t)$ is the velocity component of the fluid in the x-direction, P the fluid pressure, τ_{xy} the stress tensor in the classical nonpolar theory, η the coefficient of couple stresses and B_0 the external magnetic field. For a non-Newtonian fluid obeying the Eyring Powell model we have

$$\tau_{xy} = \mu \frac{\partial u}{\partial y} + \frac{1}{\beta}\sinh^{-1}\left(\frac{1}{c}\frac{\partial u}{\partial y}\right). \tag{3}$$

We take the first and second order approximation of the $\sinh^{-1}$ function:

$$\sinh^{-1}\left(\frac{1}{c}\frac{\partial u}{\partial y}\right) \cong \frac{1}{c}\frac{\partial u}{\partial y} - \frac{1}{6}\left(\frac{1}{c}\frac{\partial u}{\partial y}\right)^3, \quad \left|\frac{1}{c}\frac{\partial u}{\partial y}\right| \prec 1. \tag{4}$$

Then (1) will be reduced to

$$\frac{\partial u}{\partial t} + V_0 \frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho\beta c}\frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho\beta c}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho}B_0^2 u - \frac{\eta}{\rho}\frac{\partial^4 u}{\partial y^4}. \tag{5}$$

The appropriate boundary and initial conditions are

$$\left.\begin{aligned} &u = 0,\ u'' = 0 && \text{at } y = 0,\\ &u = 0,\ u'' = 0 && \text{at } y = L,\\ &0 < y < 1,\ u = V_0 \sin\left(\frac{\pi y}{L}\right),\ t \le 0 && \end{aligned}\right\}, \tag{6}$$

where a dash means differentiation with respect to y.

Let us introduce non-dimensional quantities as follows:

$$x^* = \frac{x}{L},\ y^* = \frac{y}{L},\ t^* = \frac{V_0}{L}t,\ u^* = \frac{u}{V_0},\ P^* = \frac{P}{\rho V_0^2}. \tag{7}$$

Substituting from (7) into (5) (* is dropped) we get

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{N^*}{R_\mathrm{e}}\frac{\partial^2 u}{\partial y^2} - D^*\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} - \frac{H_\mathrm{a}^2}{R_\mathrm{e}}u - \frac{1}{a^2 R_\mathrm{e}}\frac{\partial^4 u}{\partial y^4}, \tag{8}$$

subjected to the boundary conditions

$$\left.\begin{aligned} &u = 0,\ u'' = 0 && \text{at } y = 0,\\ &u = 0,\ u'' = 0 && \text{at } y = 1,\\ &0 < y < 1,\ u = \sin(\pi y),\ t \le 0 && \end{aligned}\right\}, \tag{9}$$

where $R_\mathrm{e} = \dfrac{V_0 L}{v} \equiv$ Reynolds number,

$H_\mathrm{a} = B_0 L\sqrt{\dfrac{\sigma}{\rho v}} \equiv$ Hartmann number, and

$$v = \frac{\mu}{\rho},\ N^* = 1 + M,\ M = \frac{1}{\mu c \rho}, \qquad D^* = \frac{V_0}{2\rho\beta c^3 L^3},\ a^2 = \frac{L^2}{h^2},\ h^2 = \frac{\eta}{\mu}. \tag{10}$$

Case (I): Pulsatile Unsteady MHD Flow in Case of the First Approximation

Suppose a pulsation pressure gradient of the form

$$-\frac{\partial P}{\partial x} = \left(\frac{\partial P}{\partial x}\right)_\mathrm{s} + \left(\frac{\partial P}{\partial x}\right)_0 e^{\mathrm{i}\omega \mathrm{t}} = P_\mathrm{s} + P_0 e^{\mathrm{i}\omega \mathrm{t}}, \tag{11}$$

where P_s and P_0 are constants. P_0 is the pulsation of the pressure parameter.

Then (8), corresponding to the first approximation in (4) is reduced to

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{N^*}{R_\mathrm{e}}\frac{\partial^2 u}{\partial y^2} - \frac{H_\mathrm{a}^2}{R_\mathrm{e}}u - \frac{1}{a^2}\frac{\partial^4 u}{\partial y^4}, \tag{12}$$

To obtain the solution of (12) we use the perturbation technique as follows:

$$u = u_0 + u_1 e^{\mathrm{i}\omega \mathrm{t}}. \tag{13}$$

Substituting (11) and (13) into (12) we get the two ordinary differential equations

$$\frac{1}{a^2}\frac{\mathrm{d}^4 u_0}{\mathrm{d}y^4} - \frac{N^*}{R_\mathrm{e}}\frac{\mathrm{d}^2 u_0}{\mathrm{d}y^2} + \frac{\mathrm{d}u_0}{\mathrm{d}y} + \frac{H_\mathrm{a}^2}{R_\mathrm{e}}u_0 = P_\mathrm{s} \tag{14}$$

and

$$\frac{1}{a^2}\frac{\mathrm{d}^4 u_1}{\mathrm{d}y^4} - \frac{N^*}{R_\mathrm{e}}\frac{\mathrm{d}^2 u_1}{\mathrm{d}y^2} + \frac{\mathrm{d}u_1}{\mathrm{d}y} + \left(\frac{H_\mathrm{a}^2}{R_\mathrm{e}} + i\omega\right)u_1 = P_0, \tag{15}$$

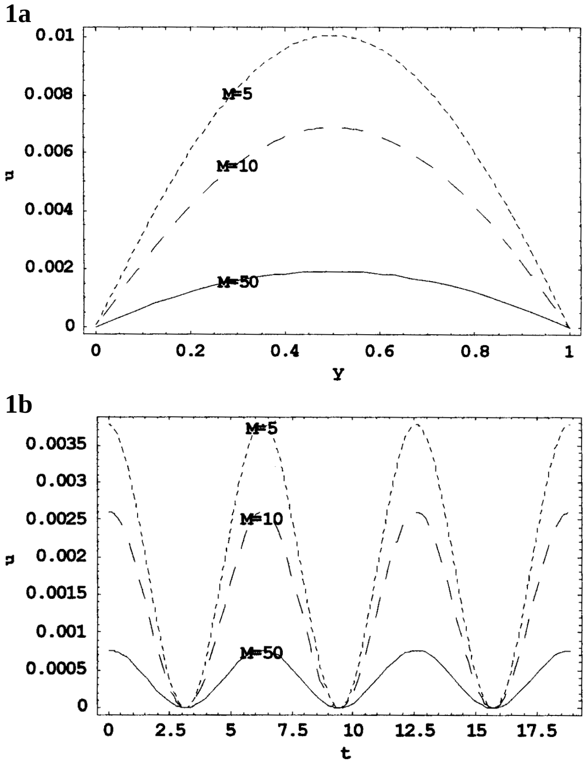


Fig. 1. Velocity distribution plotted against position (a) and time (b) in case I at $a = 0.5$, $R_\mathrm{e} = 0.1$, $H_\mathrm{a} = 3$ and $P_0 = 5$.

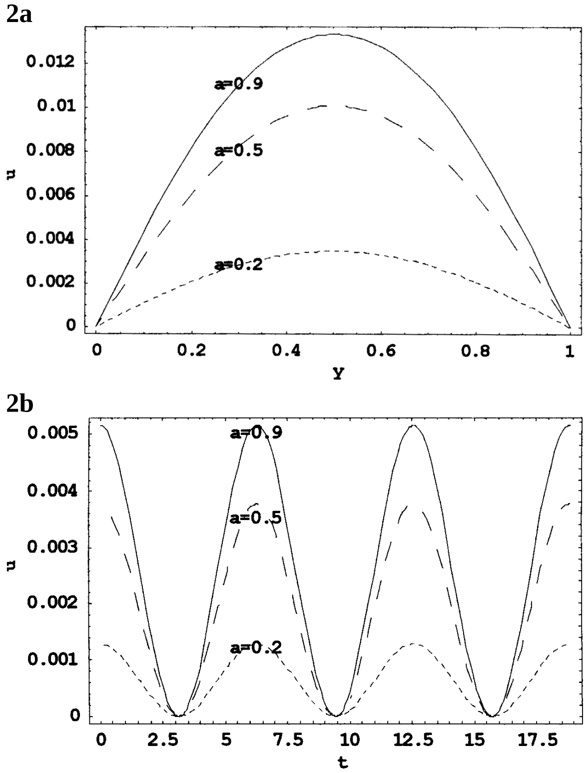


Fig. 2. Velocity distribution plotted against position (a) and time (b) in case I at $M = 5$, $R_\mathrm{e} = 0.1$, $H_\mathrm{a} = 3$ and $P_0 = 5$.

subjected to the boundary conditions

$$\left.\begin{array}{ll} u_0 = u_1 = 0,\ u_0'' = u_1'' = 0 & \text{at}\, y = 0 \\ u_0 = u_1 = 0,\ u_0'' = u_1'' = 0 & \text{at}\, y = 1 \end{array}\right\}. \quad (16)$$

The solutions of (14) and (15) under the boundary conditions (16) are evaluated by using the Mathematica program (version 4) for different values of the parameters a, M, H_a, R_e and P_0. The effects of these parameters are discussed graphically. Actually, because of the size of the solutions we will only show the graphical representations of these solutions.

Case (II): Constant Pressure Gradient in Case of the Second Approximation

To obtain the numerical solution of (8) in case of the second approximation of the Eyring Powell model we use the finite difference technique [6,7,10]. Equation (8) can be written as

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \left(\frac{N^*}{R_\mathrm{e}} - D^*\left(\frac{\partial u}{\partial y}\right)^2\right)\frac{\partial^2 u}{\partial y^2}$$

$$-\frac{H_\mathrm{a}^2}{R_\mathrm{e}}u - \frac{1}{a^2 R_\mathrm{e}}\frac{\partial^4 u}{\partial y^4}, \quad (17)$$

with the initial and boundary conditions

$$\left.\begin{array}{lll} 0 < y < 1, & u = \sin \pi y & \text{for}\, t \le 0 \\ y = 0, 1, & u = 0, u'' = 0 & \text{for}\, t > 0 \end{array}\right\}, \quad (18)$$

where M and D^* represent the non-Newtonian effects, which vanish in Newtonian flows. The numerical solution requires three initial conditions for u, y and t. The initial velocity profile may only be specified exactly if a complete analysis of the entrance region is available. However its distribution in a very little distance from the axes is assumed to be uniform.

Numerical Procedure

The flow field region is defined as an infinite strip bounded by the two parallel porous plates at $y = 0$ and $y = 1$. Under the finite difference approximation

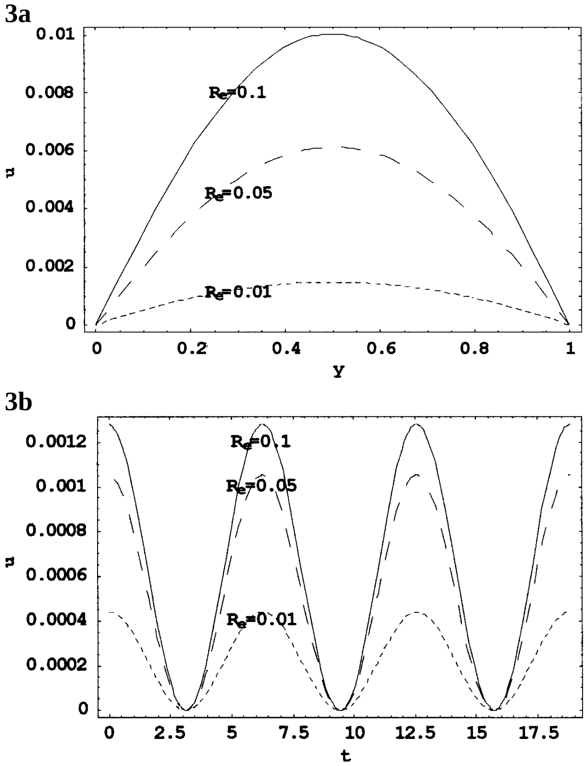


Fig. 3. Velocity distribution plotted against position (a) and time (b) in case I at $M=5$, $a=0.5$, $H_\mathrm{a}=3$ and $P_0=5$.

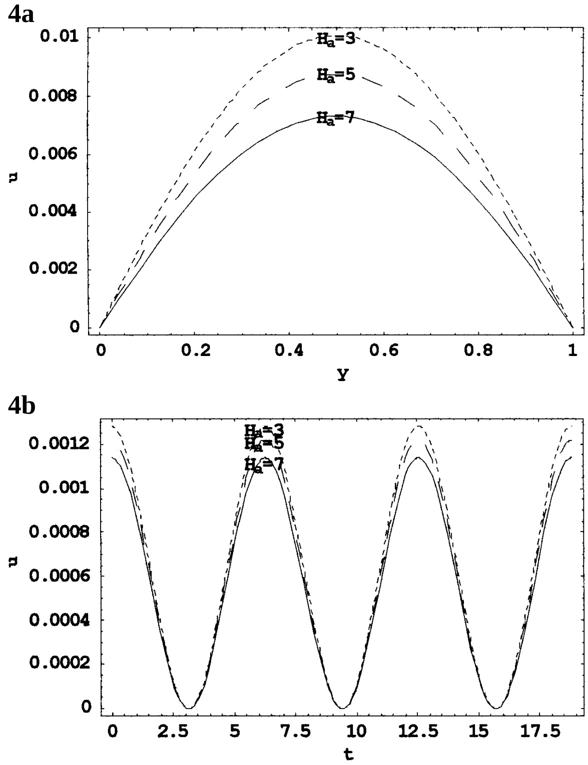


Fig. 4. Velocity distribution plotted against position (a) and time (b) in case I at $M=5$, $R_\mathrm{e}=0.1$, $a=0.5$ and $P_0=5$.

$$u_{i,j}=u(i\Delta y, j\Delta t),$$

$$\left.\frac{\partial u}{\partial t}\right|_{i\Delta y, j\Delta t}=\frac{u_{i,j+1}-u_{i,j}}{\Delta t},\quad \left.\frac{\partial u}{\partial y}\right|_{i\Delta y, j\Delta t}=\frac{u_{i+1,j}-u_{i,j}}{\Delta y},$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{i\Delta y, j\Delta t}=\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{(\Delta y)^2}, \tag{19}$$

$$\left.\frac{\partial^4 u}{\partial y^4}\right|_{i\Delta y, j\Delta t}=\frac{4}{3}\big[(u_{i+2,j}-4u_{i+1,j}+6u_{i,j} - 4u_{i-1,j}+u_{i-2,j})\big/(\Delta y)^4\big]$$

(17) becomes

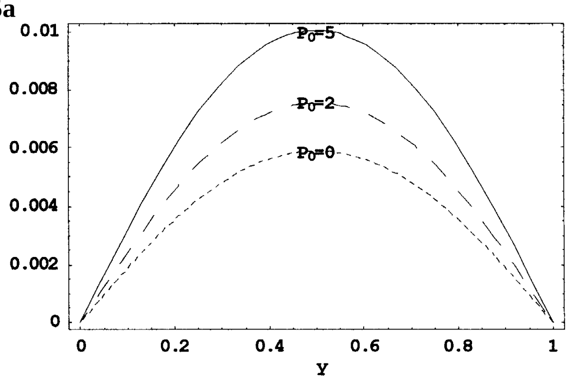
$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t}+\frac{u_{i+1,j}-u_{i,j}}{\Delta y}= K+\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{(\Delta y)^2}\left\{\frac{N^*}{R_\mathrm{e}}-D^*\left(\frac{u_{i+1,j}-u_{i,j}}{\Delta y}\right)^2\right\} -\frac{H_\mathrm{a}^2}{R_\mathrm{e}}u_{i,j}-\frac{4}{3a^2R_\mathrm{e}}\big[(u_{i+2,j}-4u_{i+1,j}+6u_{i,j} - 4u_{i-1,j}+u_{i-2,j})\big/(\Delta y)^4\big], \tag{20}$$

which can be written in the form

$$\begin{aligned} u_{i,j+1}=\Delta t &\cdot\Bigg\{K+\left(\frac{1}{\Delta t}+\frac{1}{\Delta y}-\frac{2N^*}{R_\mathrm{e}(\Delta y)^2}-\frac{H_\mathrm{a}^2}{R_\mathrm{e}}-\frac{8}{a^2R_\mathrm{e}(\Delta y)^4}\right)u_{i,j}\\ &+\left(\frac{N^*}{R_\mathrm{e}(\Delta y)^2}-\frac{1}{\Delta y}+\frac{16}{3a^2R_\mathrm{e}(\Delta y)^4}\right)u_{i+1,j}\\ &+\left(\frac{N^*}{R_\mathrm{e}(\Delta y)^2}+\frac{16}{3a^2R_\mathrm{e}(\Delta y)^4}\right)u_{i-1,j}\\ &-\frac{4}{3a^2R_\mathrm{e}(\Delta y)^4}u_{i+2,j}-\frac{4}{3a^2R_\mathrm{e}(\Delta y)^4}u_{i-2,j}\\ &-\Big[(u_{i+1,j})^3-4u_{i,j}(u_{i+1,j})^2-3(u_{i,j})^2u_{i+1,j}\\ &\quad-2(u_{i,j})^3+(u_{i+1,j})^2u_{i-1,j}-2u_{i,j}u_{i+1,j}u_{i-1,j}\\ &\quad+(u_{i,j})^2u_{i-1,j}\Big]\frac{D^*}{(\Delta y)^4}\Bigg\}, \end{aligned} \tag{21}$$

where K is the constant pressure gradient.

5a



5b

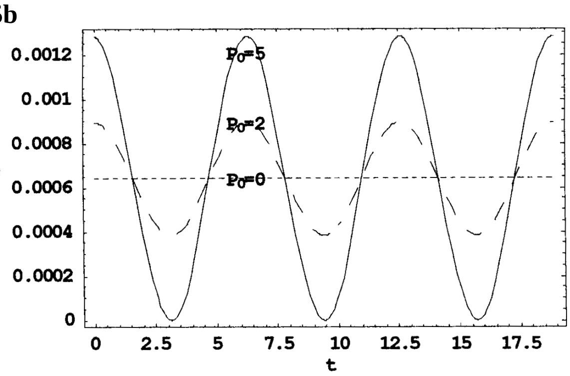


Fig. 5. Velocity distribution plotted against position (a) and time (b) in case I at $M = 5$, $R_\mathrm{e} = 0.1$, $H_\mathrm{a} = 3$ and $a = 0.5$.

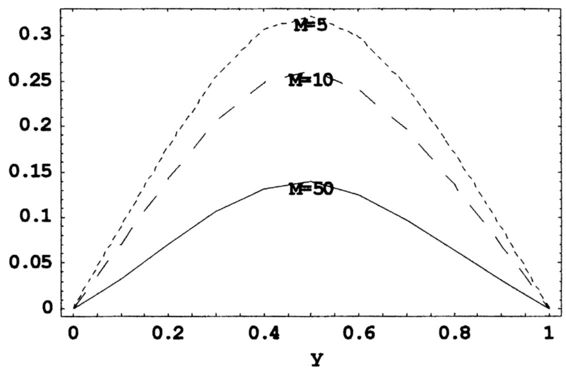


Fig. 6. Velocity distribution plotted against position (a) and time (b) in case II at $a = 0.5$, $R_\mathrm{e} = 0.1$, $H_\mathrm{a} = 3$ and $P_0 = 5$, $D^* = 0.05$.

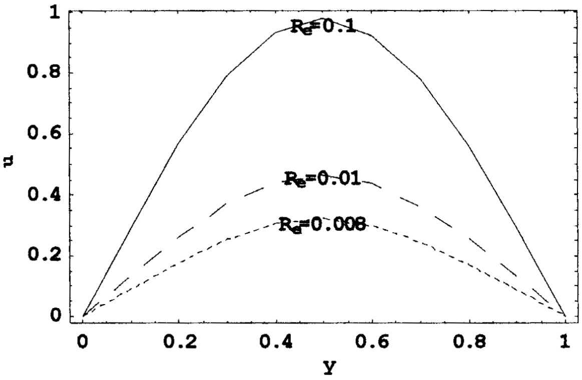


Fig. 7. Velocity distribution plotted against position (a) and time (b) in case II at $M = 5$, $a = 0.5$, $H_\mathrm{a} = 3$ and $P_0 = 5$, $D^* = 0.05$.

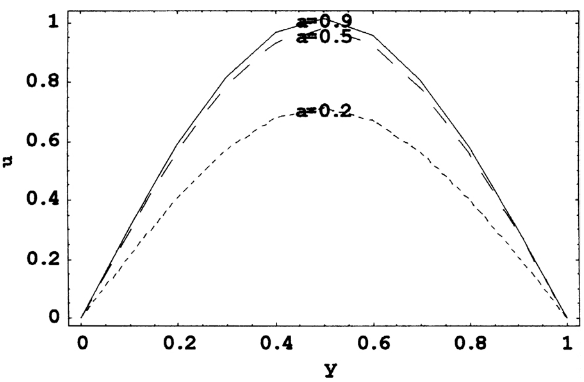


Fig. 8. Velocity distribution plotted against position (a) and time (b) in case II at $M = 5$, $R_\mathrm{e} = 0.1$, $H_\mathrm{a} = 3$ and $P_0 = 5$, $D^* = 0.05$.

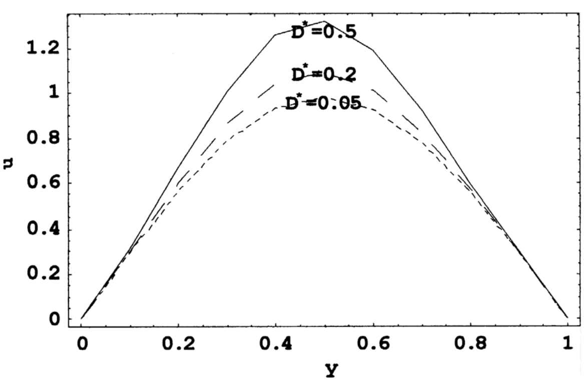


Fig. 9. Velocity distribution plotted against position (a) and time (b) in case II at $M = 5$, $R_\mathrm{e} = 0.1$, $H_\mathrm{a} = 3$ and $P_0 = 5$, $a = 0.05$.

Discussion of the Results

The effect of different parameters entering the problem is illustrated graphically in Figures (1 – 9).

For case I:

It is found from Figs. 1 (a) and (b) that the velocity decreases with increasing M, which is the first approximation parameter of the non-Newtonian fluid effect. Also, from Figs. 2 (a) and (b) we can see that the velocity increases with increasing a^2 which is the inverse of the couple stresses parameter, which shows that the velocity of the fluid decreases with increasing of couple stresses. Also we can see from Figs. 3 (a) and (b) that the velocity decreases with increasing Reynolds number R_e for constant Hartmann number H_a. It is also found that the velocity decreases with increasing H_a for constant R_e as we can see from Figs. 4 (a) and (b). From Fig. 5(a) we can see that the velocity increases with increasing pulsation of the pressure parameter P_0 for constant time, but from Fig. 5(b) we can see that this effect changes from increasing to decreasing with the change of time.

For case II:

It is found that the velocity decreases with increasing M, see Figure 6. Also Fig. 7 shows that the velocity decreases with decreasing Reynolds number R_e. The velocity increases with increasing a^2 which can be seen from Fig. 8, i.e. the velocity decreases with increasing couple stresses. Finally, we can see from Fig. 9 that the velocity increases with increasing second approximation parameter D^*.

Conclusions

In this paper we have studied the effect of couple stresses on an unsteady MHD non-Newtonian flow between two parallel fixed porous plates under a uniform external magnetic field in models like the Eyring Powel model. We conclude that the flow is damping with increasing effect of couple stresses. This result may be very useful in many cases, like the discussion of some diseases of the blood. Couple stresses, for the model under consideration, depend on vorticity gradients. Since vorticity gradients are known to be large in hydromagnetic flows of nonpolar fluids, couple stress effects may be expected to be large in electrically conducting polar fluids also.These effects – as we deduce in the paper – are causing a remarkable degradation in the flow of the fluid, which may be useful in studying the blood flow in the arteries, especially the defective ones-for any reasons. We can obtain a good way of decreasing this flow which leads to decrease the blood pressure and any related diseases.

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